

نجم الرفيع بواسطة
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Calculus 2
~~Second~~
First

Palestine Technical University
Department of Applied Mathematics.

Calculus II
First Exam

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Duration: 70 min.

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Q1. (30 points) Choose the correct answer.

(1) Let f and g be two differentiable functions. If g is the inverse of f and if $g(-2) = 5$ and

$f'(5) = \frac{-1}{2}$, then $g'(-2) =$

(a) 2

(c) $\frac{-1}{2}$

(d) None of the above

(2) The value of $\lim_{x \rightarrow \infty} \left(x \tan^{-1} \frac{2}{x} \right) =$

(a) $1/2$

(b) 1

(c) 2

(d) ∞

(3) The value of $2^{\log_4 9} =$

(a) 3

(b) 9

(c) $9/4$

(d) $\frac{\ln 9}{\ln 4}$

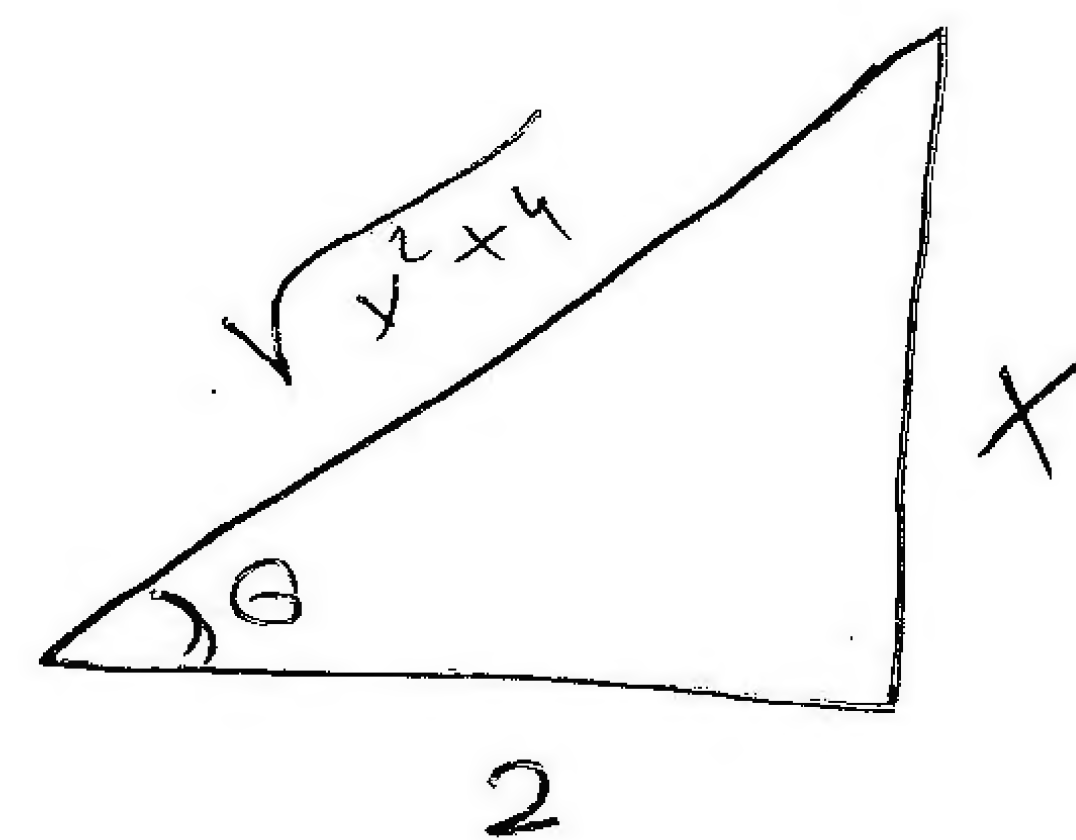
(4) The value of $\sec \left(\tan^{-1} \frac{x}{2} \right) =$

(a) $\frac{x}{\sqrt{x^2 + 4}}$

(b) $\frac{2}{x}$

(c) $\frac{\sqrt{x^2 + 4}}{2}$

(d) $\frac{2}{\sqrt{x^2 + 4}}$



18
22
20
19

$$\sqrt{\ln x} \Big|_2^{16} = \sqrt{\ln 16} - \sqrt{\ln 2}$$

$$(\ln 16)^{\frac{1}{2}} - (\ln 2)^{\frac{1}{2}}$$

$$\frac{1}{2} \ln 16 - \frac{1}{2} \ln 2$$

$$\frac{1}{2} \ln 2^4 - \frac{1}{2} \ln 2$$

$$2 \ln 2 - \frac{1}{2} \ln 2 = \frac{3}{2} \ln 2$$

(a)

(5) If $y = \tan^{-1}(\ln x)$, then $\frac{dy}{dx}$ at $x = e$ is

- (a) $\frac{\pi}{4}$ (b) $\frac{1}{1+e^2}$
(c) $\frac{1}{4e}$ (d) $\frac{1}{2e}$

$$y = \tan^{-1}(\ln x)$$

$$y' = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

$$\frac{1}{1+1} \cdot \frac{1}{e} = \frac{1}{2} \cdot \frac{1}{e} = \frac{1}{2e}$$

(6) The value of the integral $\int_0^2 \frac{dt}{8+2t^2} =$

- (a) $\frac{\pi}{16}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{12}$

(d) π

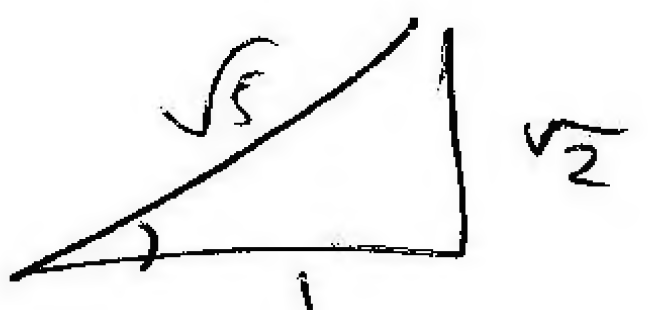
$$\frac{1}{\sqrt{8} + \sqrt{2}t} = \frac{1}{\sqrt{8} + \sqrt{2}t} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{16} + 2t} = \frac{1}{4 + 2t}$$

$$\tan^{-1} \frac{\sqrt{2}t}{\sqrt{8}} = \tan^{-1} \frac{\sqrt{2}t}{2\sqrt{2}} = \tan^{-1} \frac{t}{2}$$

$$\frac{1}{2} \int_0^2 \frac{du}{4+u^2} = \frac{1}{2} \left[\tan^{-1} \frac{u}{2} \right]_0^2 = \frac{1}{2} \left[\tan^{-1} 1 - \tan^{-1} 0 \right] = \frac{1}{2} \left[\frac{\pi}{4} - 0 \right] = \frac{\pi}{8}$$

$$u = 2t$$

$$du = 2 dt$$



(7) The value of the integral $\int_2^{16} \frac{dx}{2x\sqrt{\ln x}} =$

- (a) $2 - \sqrt{\ln 2}$ (b) $\sqrt{\ln 2}$
(c) $\ln 2 - \ln 5$ (d) $3 \ln 2$

$$\frac{1}{2} \int \frac{1}{u} \cdot \frac{1}{2x\sqrt{u}} du = \frac{1}{4} \int \frac{1}{u\sqrt{u}} du = \frac{1}{4} \int u^{-3/2} du = \frac{1}{4} \left[-2u^{-1/2} \right] = -\frac{1}{2\sqrt{u}}$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{u}{\sqrt{8}} \right) = \frac{1}{2} \tan^{-1} \left(\frac{2t}{\sqrt{8}} \right)$$

$$\frac{1}{2} \left[\tan^{-1} \left(\frac{u}{\sqrt{8}} \right) - \tan^{-1} (0) \right]$$

$$\frac{1}{2} \left[\tan^{-1} (\sqrt{2}) - 0 \right] = \frac{1}{2} \left[\frac{\pi}{4} \right] = \frac{\pi}{8}$$

(8) Let $f(x) = \frac{e^x + e^{-x}}{2}$. The value of the integral $\int_1^2 \frac{f(\ln(x))}{x} dx =$

- (a) $3/4$ (b) $1/2$
(c) $1/4$ (d) 1

$$\frac{1}{2} \int \frac{1}{u} \cdot \frac{1}{2x\sqrt{u}} du = \frac{1}{4} \int \frac{1}{u\sqrt{u}} du = \frac{1}{4} \int u^{-3/2} du = \frac{1}{4} \left[-2u^{-1/2} \right] = -\frac{1}{2\sqrt{u}}$$

$$\frac{1}{2} \int \frac{1}{u} \cdot \frac{1}{2x\sqrt{u}} du = \frac{1}{4} \int \frac{1}{u\sqrt{u}} du = \frac{1}{4} \int u^{-3/2} du = \frac{1}{4} \left[-2u^{-1/2} \right] = -\frac{1}{2\sqrt{u}}$$

$$\frac{1}{2} \int \frac{1}{u} \cdot \frac{1}{2x\sqrt{u}} du = \frac{1}{4} \int \frac{1}{u\sqrt{u}} du = \frac{1}{4} \int u^{-3/2} du = \frac{1}{4} \left[-2u^{-1/2} \right] = -\frac{1}{2\sqrt{u}}$$

(9) If $f(x) = \ln(\sec^2 x)$, then $f'(x) =$

- (a) $\ln(\tan x)$ (b) $2 \sec x$
(c) $\ln(2 \sec x)$ (d) $2 \tan x$

$$\frac{1}{\sec^2 x} \cdot (2 \sec x) \cdot (\sec x \tan x) = \frac{2 \sec x \tan x}{\sec^2 x} = \frac{2 \tan x}{\sec x} = 2 \sin x$$

$$\frac{x^2+1}{2x} = \frac{x}{2} + \frac{1}{2x}$$

10. $\sin^{-1} \sin \frac{3\pi}{4} =$

(a) $-\frac{3\pi}{4}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $-\frac{\pi}{4}$

$\frac{1}{35}$

$\sin 135^\circ$
 $180 - 135 = 45$
 $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\sin 45^\circ$

$\frac{1}{\sqrt{2}}$

$$\ln x = u$$

$$du = \frac{1}{x} dx$$

$$\int \frac{du}{2x\sqrt{u}} = \frac{1}{2} \int \frac{du}{u\sqrt{u}} = \frac{1}{2} \int u^{-3/2} du = \frac{1}{2} \left[-2u^{-1/2} \right] = -\frac{1}{\sqrt{u}}$$

$$= \frac{1}{2} \int \frac{du}{u\sqrt{u}} = \frac{1}{2} \int u^{-3/2} du = \frac{1}{2} \left[-2u^{-1/2} \right] = -\frac{1}{\sqrt{u}}$$

$$\frac{1}{2} \int \frac{u^{-1/2} + 1}{-1/2 + 1} du = \frac{1}{2} \int \frac{u^{-1/2} + 1}{1/2} du = \int (u^{-1/2} + 1) du = 2\sqrt{u} + u$$

$$\sqrt{\ln x} \Big|_2^{16} = \sqrt{\ln 16} - \sqrt{\ln 2}$$

$$(\ln 16)^{\frac{1}{2}} - (\ln 2)^{\frac{1}{2}}$$

$$\frac{1}{2} \ln 2^4 - \frac{1}{2} \ln 2$$

$$= \sqrt{\ln 16} - \sqrt{\ln 2}$$

Q2. (23 points) Let R be the region bounded by $y = x^2$ and $y = 4x - x^2$.

Find the volume of the solid of revolution generated when R is revolved around

(a) The x-axis.

(13 pts.)

We'll use washer method

$$\text{inner} = x^2$$

$$\text{outer} = 4x - x^2$$

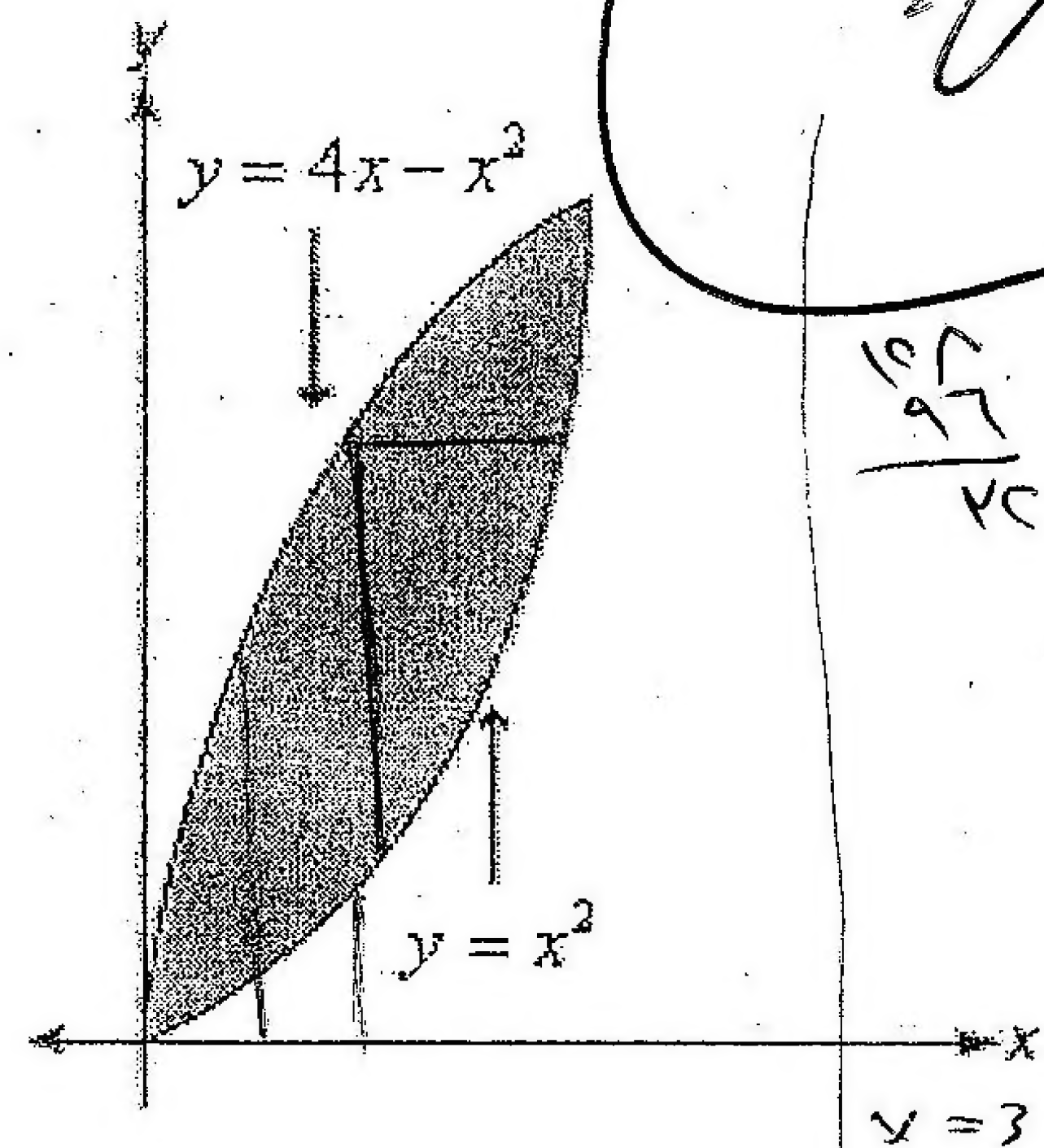
$$r^2(x) = (x^2)^2 = x^4$$

$$R^2(x) = (4x - x^2)^2 = x^4 - 8x^3 + 16x^2$$

$$\text{Point of intersection: } 4x - x^2 = x^2$$

$$4x - 2x^2 = 0 \Rightarrow 2x(2 - x) = 0$$

$$x = 0, x = 2$$



$$\begin{aligned} V &= \pi \int_0^2 (x^4 - 8x^3 + 16x^2) - x^4 dx = \pi \int_0^2 (-8x^3 + 16x^2) dx = \pi \left[-\frac{8x^4}{4} + \frac{16x^3}{3} \right]_0^2 \\ &= \pi \left[-2x^4 + \frac{16x^3}{3} \right]_0^2 = \pi \left[-2(16) + \frac{16(8)}{3} \right] - [0] \\ &= \pi \left[-32 + \frac{128}{3} \right] = \pi \left[-\frac{96}{3} + \frac{128}{3} \right] = \pi \left[\frac{32}{3} \right] = \frac{32}{3} \pi \text{ units} \end{aligned}$$

(b) The line $x = 3$.

(10 pts.)

We'll use shell method

$$\text{radius} = (3 - x)$$

$$\text{height} = 4x - x^2 - x^2 = 4x - 2x^2$$

$$\text{Thickness} = dx$$

$$\begin{aligned} V &= 2\pi \int_0^2 (3 - x)(4x - 2x^2) dx = 2\pi \int_0^2 (12x - 6x^2 - 4x^2 + 2x^3) dx \\ &= 2\pi \int_0^2 (12x - 10x^2 + 2x^3) dx = 2\pi \left[\frac{12x^2}{2} - \frac{10x^3}{3} + \frac{2x^4}{4} \right]_0^2 \\ &= 2\pi \left[6x^2 - \frac{10}{3}x^3 + \frac{x^4}{2} \right]_0^2 \\ &= 2\pi \left[24 - \frac{10}{3}(8) + \frac{16}{2} \right] - [0] \\ &= 2\pi \left[24 + 8 - \frac{80}{3} \right] = 2\pi \left[32 - \frac{80}{3} \right] \\ &= 2\pi \left[\frac{96}{3} - \frac{80}{3} \right] = 2\pi \left[\frac{16}{3} \right] = \frac{32}{3} \pi \text{ units} \end{aligned}$$

Q3. (23 points)

(a) Use logarithmic differentiation to find the derivative of

(15 pts.)

$$y = \sqrt[3]{\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6}}$$

$$\ln y = \ln \left(\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6} \right)^{\frac{1}{3}}$$

$$\ln y = \frac{1}{3} \left[\ln x + \ln(x+1)^9 + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)^6 \right]$$

$$\ln y = \frac{1}{3} \left[\ln x + 9 \ln(x+1) + \ln(x-2) - \ln(x^2+1) - 6 \ln(2x+3) \right]$$

$$\frac{1}{y} \cdot y' = \frac{1}{3} \left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{6(2)}{2x+3} \right]$$

$$\therefore y' = \frac{1}{3} \left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{12}{2x+3} \right] [y]$$

$$\therefore y' = \frac{1}{3} \left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{12}{2x+3} \right] \left[\sqrt[3]{\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6}} \right]$$

(b) Set up an integral to find the arc length of the graph $f(x) = 2e^{3x}$ from $x = 0$ to $x = e$

(8 pts.)

$$L = \int_0^e \sqrt{1 + f'(x)^2} dx$$

$$f(x) = 2e^{3x}$$

$$f'(x) = (2)(3)e^{3x} = 6e^{3x}$$

$$\therefore L = \int_0^e \sqrt{1 + [6e^{3x}]^2} dx$$

$$7e^{3x}$$

20

Q4. (24 points)

(a) Evaluate $\int 3^x 2^{2x} dx$

(6 pts.)

$$\int 3^x 2^{2x} dx = \int 3^x 4^x dx = \int 12^x dx$$

$$= \frac{1}{\ln 12} \cdot 12^x + C = \boxed{\frac{12^x}{\ln 12} + C}$$

6

~~$x^2 + 4x + 4$~~
 ~~$(x+2)^2$~~

(b) Evaluate $\int \frac{dx}{\sqrt{-x^2 + 4x - 3}}$

(10 pts.)

$$\int \frac{dx}{\sqrt{(x^2 - 4x - 4) - 3 + 4}} = \int \frac{dx}{\sqrt{-(x+2)^2 + 1}} = \int \frac{dx}{\sqrt{1 - (x+2)^2}}$$

Let $u = x+2 \Rightarrow du = dx$

~~$\int \frac{du}{\sqrt{1-u^2}}$~~

$$I = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$$

$$= \sin^{-1}(x+2) + C$$

10

The root

(c) If $f(x) = (2x-3)^3$. Show that the function f has an inverse. Find $f^{-1}(x)$.

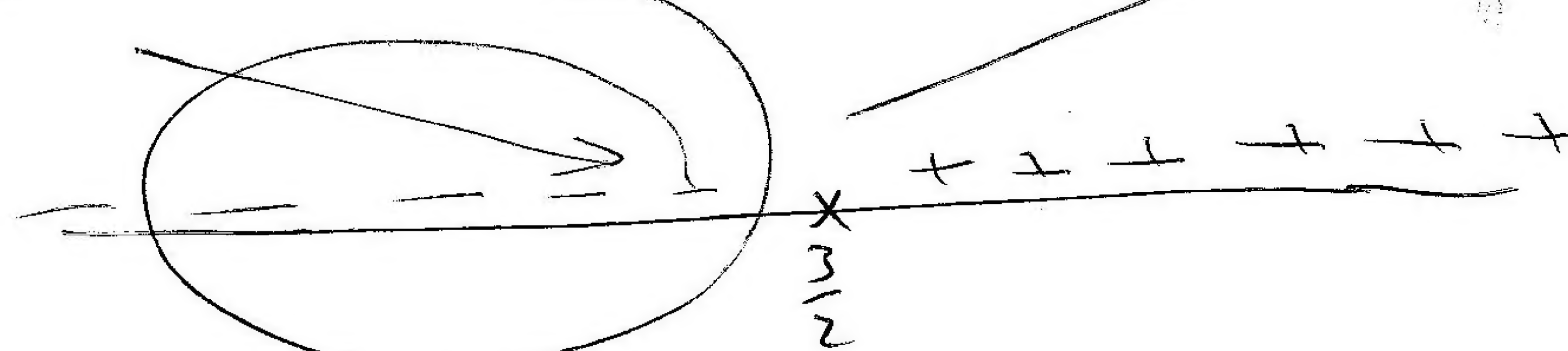
(8 pts.)

$$f(x) = (2x-3)^3 \quad f(x) = 0 \Rightarrow (2x-3)^3 = 0 \Rightarrow (2x-3) = 0 \Rightarrow x = \boxed{\frac{3}{2}}$$

$$f'(x) = 3(2x-3)^2(2) = \boxed{6(2x-3)^2}$$

$$f'(x) = 0 \Rightarrow 6(2x-3)^2 = 0 \Rightarrow (2x-3)^2 = 0$$

$$2x-3 = 0 \Rightarrow x = \boxed{\frac{3}{2}}$$



We notice that the function after $x = \frac{3}{2}$ is ~~increasing~~ always increasing, and if function increasing, we can conclude that the function has an inverse [and pass the horizontal line test].